DONATIONS.

Comptes Rendus Hebdomadaires des Séances de l'Academie des Sciences. Par. MM. les Sécretaires Perpetuels. Second Semestre, Nos. 26 and 27. Presented by the Academy.

Memorie della Reale Accademia delle Scienze di Torino. Tome XL. Presented by the Academy.

Transactions of the Society of Civil Engineers. Vol. II. Presented by the Society.

Report of a Committee of the Royal Society, on the Propriety of recommending to her Majesty's Government, the Establishment of fixed Magnetic Observatories, and the Equipment of a Naval Expedition for Magnetic Observations in the Antarctic Seas; together with the Resolutions adopted on that Report by the Council of the Royal Society. Presented by the Society.

Rough Sketches, intended to aid in developing the Natural History of the Seals, (Phocidæ) of the British Islands. By R. Ball, Esq. Presented by the Author.

The Expediency and Facility of establishing Metrological and Monetary Systems throughout India, on a scientific and permanent Basis, grounded on an analytical Review of the Weights, Measures, and Coins of India, &c. &c. By Captain T. B. Jervis. Presented by the Author.

The Indian Review, and Journal of Foreign Science and the Arts. Edited by Frederick Corbyn, Esq. Vol. II. Presented by the Editor.

February 11.

SIR WM. R. HAMILTON, A. M., President, in the Chair.

The Rev. Robert Vickers Dixon, F.T.C., was elected a Member of the Academy.

Mr. George Smith presented to the Academy an original portrait of General Vallancey, by Chinnery.

The thanks of the Academy were voted to Mr. Smith for his valuable donation.

Mr. William Bald, civil engineer, read a paper, containan account of his models of the Island of Achil, Clare Island, and the south western district of Mayo, which comprises the greater part of the barony of Murrisk, and a portion of the barony of Burrishoole.

After an introduction, in which the author calls attention to modelling, as the best mode of representing the rise and fall of ground, and makes some remarks on the undeserved neglect which the subject has hitherto met with in this country, he proceeds to state, that his model of Clare Island was made on a scale of eight inches to the Irish mile; and that the models of Achil Island, and Murrisk Barony, were on scales of four inches to the English mile. The vertical scales were the same as the horizontal. The model of the barony of Murrisk is at present deposited in the house of the Royal Dublin Society. The original model of Clare Island is in the possession of the Royal Society of Edinburgh; and the model of Achil has been deposited in the Museum of the College of Edinburgh. The model of Murrisk represents the area of a country containing nearly two hundred square miles; that of Achil represents a country containing about fifty-eight square miles; and Clare Island is about four English miles long, by two and a quarter in its greatest breadth. These models were constructed with a composition of putty, white lead, and cork. The paper gives an account in detail of the mode of their construction.

In ascertaining the levels of the country, Mr. Bald recommends that lines of equal level be adopted, and also

transverse sections made. He states that neither Ben Nevis in Scotland, Snowdon in Wales, nor Macgillycuddie's Reeks in Ireland, have had their heights ascertained by actual levelling; and he observes, that were these heights accurately determined, further knowledge might then be obtained regarding refraction, and the measurement of altitudes by the barometer. A small map of the island of Inish Turk, on which are delineated lines of equal level, accompanies the paper.

In conclusion, Mr. Bald observes, that a model of a country in the hands of the topographic, military, civil, or mining engineer, could be applied to a variety of useful purposes; and particularly that it would enable young men to shade accurately topographic maps, a thing that has not yet, to his knowledge, been systematically attended to in any of the institutions of Great Britain or Ireland.

Rev. H. Lloyd, V. P., read a paper "on the relative Position of three Magnets, in a Magnetical Observatory."

It is a problem of much importance, in connexion with the arrangement of a magnetical observatory, to determine the relative position of the magnetical instruments in such a manner, that their mutual action may be either absolutely null, or at least, readily calculable. Such was stated by the author to be the object of the investigation now laid before the Academy.

In the case of *two* horizontal magnets, one of which (intended for observations of *declination*) is in the magnetic meridian, and the other (used for observations of *horizontal intensity*) is in the perpendicular plane, there is nothing to compensate the action of each magnet on the other. The best thing that can be done in this case, is to determine the position of the second magnet in such a manner, that the direction of its action on the first shall coincide with *the magnetic meridian*. In such case, the position of the first

magnet will be undisturbed by the second, so as to give the *absolute declination* truly; and, as to the *variations* of the declination, it is manifest that they will be thereby increased or diminished in a *given ratio;* so that the true variations will be obtained by simply altering the coefficient of the scale. When the above-mentioned condition is introduced into the equation which determines the direction of the resultant force exerted by one magnet on another, (the length of the magnets being supposed small in comparison with the distance between them,) we find, for the azimuth of the line connecting the two magnets, referred to the magnetic meridian,

$$\text{arc}\left(\tan = \frac{1}{\sqrt{2}}\right) = 35^\circ\ 16'.$$

This result has been already obtained by Gauss and Weber.

It is manifest that, in this case, the action of the first magnet on the second will not take place, either in the magnetic meridian, or in the plane perpendicular to it; so that the second magnet is necessarily disturbed. With two magnets, accordingly, it is impossible to avoid the effects of mutual action. The case is different, however, when a *third* magnet is introduced. It is then possible to annul completely all action, with the exception of that exerted on the third magnet by the first and second; and this, in the case under consideration, is destroyed by the nature of the suspension.

The third magnet about to be employed in the Dublin Observatory is intended for the observation of the *vertical component of the magnetic force.* It is a bar supported on knife-edges, capable of motion in a vertical plane, and brought into the horizontal position by means of a weight. The three instruments being in the same horizontal plane, it is manifest that the action of the first and second on the third must take place in that plane; and this action can have no effect in disturbing the magnet, its motion being

confined to the vertical plane. It is only necessary to consider, therefore, the action on the first and second magnets.

The author then proceeded to the conditions of equilibrium of these actions, which were expressed by four equations, containing four arbitrary angles; so that this equilibrium is, in general, attainable, by suitably determining the position of the three magnets, whatever be their relative intensities.

In practice, however, it will seldom happen, that we can regard as arbitrary all the four angles which enter these equations, one or more of them being, in general, determined by some circumstance connected with the locality. In such case the complete destruction of all action is no longer possible; and we must look for some other solution of the problem of mutual interference.

Next to the complete destruction of all action, the most desirable course is to give to the resultant action such a direction, that its effect may be readily computed and allowed for. In the case of the declination bar, it is easily seen that this direction is the magnetic meridian itself; the *mean* position of the bar being thereby *unaltered,* and the *variations* of its position only increased or diminished in a given ratio. By means of a simple investigation it is shown, that the same thing is true of the horizontal intensity bar; and that, in order that the variations of *declination* may not be mixed up with those of *force,* the resultant force exerted upon this magnet by the other two must lie in the magnetic meridian. The problem, therefore, is reduced to this:—to determine the position of the three magnets A, B, and C, in such a manner, that the resultant actions exerted upon A and B, respectively, by the other two, shall lie in the magnetic meridian.

The solution of this problem was shown by the author to be contained in the two following equations:

$$\sin^3(\alpha+\beta)\,[3\sin(u+v-2\alpha)+\sin(u-v)] \\ +2q\sin^3\beta\,(3\sin^2 u-1)=0.$$

$$\sin^3(\alpha+\beta)\,[3\sin(u+v+2\beta)+\sin(u-v)] \\ +6p\sin^3\alpha.\,\sin u\cos u=0.$$

in which α and β denote the angles BAC and ABC, of the triangle formed by the lines joining the three magnets; u and v, the angles which the directions of the magnets, A and C, make with the line AB; and p and q the ratios of the forces of the magnets A and B to that of the third magnet C, at the unit of distance.

The paper concluded with the application of the formulæ to some remarkable cases,—as, when the three magnets are in the same right line; when the line joining two of them is in the magnetic meridian, or perpendicular to it; &c.

The Chair having been taken, *pro tempore*, by his Grace the Archbishop of Dublin, V. P., the President continued his account of his researches in the theory of light.

As a specimen of the problems which he had lately considered and resolved, the following question was stated:— An indefinite series of equal and equally distant particles, $\ldots\, m_{-1}, m_0, m_1, \ldots$, situated in the axis of x, at the points $\ldots -1, 0, +1, \ldots$, being supposed to receive, at the time 0, any very small transversal displacements $\ldots y_{-1,0}, y_{0,0}, y_{1,0}, \ldots$, and any very small transversal velocities $\ldots y'_{-1,0}, y'_{0,0}, y'_{1,0}, \ldots$, it is required to determine their displacements $\ldots y_{-1,t}, y_{0,t}, y_{1,t}, \ldots$ for any other time t; each particle being supposed to attract the one which immediately precedes or follows it in the series, with an energy $= a^2$; and to have no sensible influence on any of the more distant particles. This problem may be considered as equivalent to that of integrating generally the equation in mixed differences,

$$y''_{x,t} = a^2 (y_{x+1,t} - 2y_{x,t} + y_{x-1,t}); \qquad (1)$$

which may also be thus written:

$$\left(\frac{d}{dt}\right)^2 y_{x,t} = \frac{(a\Delta_x)^2}{1+\Delta_x} y_{x,t}. \qquad (1)$$

The general integral required, may be thus written:

$$y_{x,t} = \left\{ 1 - \frac{a^2 \Delta_x^2}{1+\Delta_x} \left(\int_0^t dt\right)^2 \right\}^{-1} (y_{x,0} + t\, y'_{x,0}); \qquad (2)$$

an expression which may be developed into the sum of two series, as follows,

$$y_{x,t} = y_{x,0} + \frac{a^2 t^2}{1.2} \Delta_x^2\, y_{x-1,0} + \frac{a^4 t^4}{1.2.3.4} \Delta_x^4\, y_{x-2,0} + \&c.$$

$$+ t y'_{x,0} + \frac{a^2 t^3}{1.2.3} \Delta_x^2\, y'_{x-1,0} + \frac{a^4 t^5}{1.2.3.4.5} \Delta_x^4\, y'_{x-2,0} + \&c.; \qquad (2)$$

and may be put under this other form,

$$y_{x,t} = \frac{2}{\pi} \Sigma_{(l)-\infty}^{\infty}\, y_{x+l,0} \int_0^{\frac{\pi}{2}} d\theta \cos(2l\theta) \cos(2\, at \sin\theta)$$

$$+ \frac{1}{a\pi} \Sigma_{(l)-\infty}^{\infty}\, y'_{x+l,0} \int_0^{\frac{\pi}{2}} d\theta \cos(2l\theta) \operatorname{cosec}\theta \sin(2\, at\sin\theta); \qquad (2)''$$

the first line of (2)′ or (2)″ expressing the effect of the initial displacements, and the second line expressing the effect of the initial velocities, for all possible suppositions respecting these initial data, or for all possible forms of the two arbitrary functions $y_{x,0}$ and $y'_{x,0}$.

Supposing now that these arbitrary forms or initial conditions are such, that

$$y_{x,0} = \eta \operatorname{vers} 2x\frac{\pi}{n}, \text{ and } y'_{x,0} = -2a\eta \sin\frac{\pi}{n} \sin 2x\frac{\pi}{n}, \qquad (3)$$

for all values of the integer x between the limits 0 and $-in$, n and i being positive and large, but finite integer numbers,

and that for all other values of x the functions $y_{x,0}$ and $y'_{x,0}$ vanish: which is equivalent to supposing that at the origin of t, and for a large number i of wave-lengths (each $= n$) behind the origin of x, the displacements and velocities of the particles are such as to agree with the following law of undulatory vibration,

$$y_{x,t} = \eta \operatorname{vers}\left(2x\frac{\pi}{n} - 2at\sin\frac{\pi}{n}\right), \tag{3}$$

but that all the other particles are, at that moment, at rest: it is required to determine the motion which will ensue, as a consequence of these initial conditions. The solution is expressed by the following formula, which is a rigorous deduction from the equation in mixed differences (1):

$$y_{x,t} = \frac{\eta}{\pi}\left(\sin\frac{\pi}{n}\right)^2 \int_0^\pi \frac{\sin in\theta}{\sin\theta}\,\frac{\cos(2x\theta + in\theta - 2at\sin\theta)}{\cos\theta - \cos\frac{\pi}{n}}\,d\theta; \tag{4}$$

an expression which tends indefinitely to become

$$y_{x,}{}^{t} = \frac{\eta}{2}\operatorname{vers}\left(2x\frac{\pi}{n} - 2at\sin\frac{\pi}{n}\right)$$

$$-\frac{\eta}{2\pi}\left(\sin\frac{\pi}{n}\right)^2 \int_0^\pi \frac{\sin(2x\theta - 2at\sin\theta)}{\sin\theta\left(\cos\theta - \cos\frac{\pi}{n}\right)}\,d\theta, \tag{4$'$}$$

as the number i increases without limit. The approximate values are discussed, which these rigorous integrals acquire, when the value of t is large. It is found that a vibration, of which the phase and the amplitude agree with the law (3)′, is propagated forward, but not backward, so as to agitate successively new and more distant particles, (and to leave successively others at rest, if i be finite,) with a velocity of progress which is expressed by $a\cos\frac{\pi}{n}$, and which is therefore less, by a finite though small amount, than the velocity of passage $a\frac{n}{\pi}\sin\frac{\pi}{n}$ of any given phase, from one vibrating

particle to another within that extent of the series which is already fully agitated. In other words, the communicated vibration does not attain a sensible amplitude, until a finite interval of time has elapsed from the moment when one should expect it to begin, judging only by the law of the propagation of phase through an indefinite series of particles, which are all in vibration already. A small disturbance, distinct from the vibration (3)′, is also propagated, backward as well as forward, with a velocity $= a$, independent of the length of the wave. And all these propagations are accompanied with a small degree of terminal diffusion, which, after a very long time, renders all the displacements insensible, if the number i, however large, be finite, that is, if the vibration be originally limited to any finite number of particles.

Dr. Apjohn read a paper by George James Knox, Esq. "on the Direction and Mode of Propagation of the electric Force traversing Media which do not undergo Electrolyzation."

In the commencement of this paper, the author details experiments, which appear to him to justify the inference, that when an electric circuit is completed through water or melted phosphorus, the current passes directly through the substance of these media, but that when, for these, a metal such as lead is substituted, the electricity moves exclusively along its surface. He next considers the source and mode of propagation of the electric force, developed in the pile, and after a brief review of the theories and experiments of Davy, Faraday, and Becquerel, arrives at the following conclusion, viz. that an electric current originates in a natural electro-inductive power of bodies when brought into contact, which affects the circumambient ether of each particle, and is continued by alternate states of induction and equilibrium; the amplitude of the oscillations of the electrical ether con-

stituting the quantity, as their rapidity constitutes the intensity, of the electrical current.

Dr. Aquilla Smith exhibited to the Academy, an ancient Irish bell, of a square form, found near Fintona in the county of Tyrone.

IT WAS RESOLVED, (on the recommendation of Council,) "that any Member of the Academy, who proposes the name of a candidate, shall give, *in writing*, the grounds on which he recommends such candidate for admission; and that such statement shall be read by the President from the chair previous to the ballot."

IT WAS RESOLVED, (on the recommendation of Council,) "that the following shall, in future, be the mode of balloting in the election of Members:—

"Members balloting are to mark with *an asterisk* the name of the candidate or candidates whom they desire to *admit;* to draw a *line through* the name of the candidate or candidates whom they wish to *reject;* and to leave the name unmarked when they do not vote at all."

DONATIONS.

Comptes Rendus Hebdomadaires des Seances de l'Academie des Sciences. Par MM. les Secretaires Perpetuels. No. I. Premier Semestre, 1839. Presented by the Academy.

An original Portrait of General Vallancey, by Chinnery. Presented by George Smith, Esq.